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UNSTEADY HYDROELASTICITY OF FLOATING PLATES

A. KOROBKIN
Lavrentyev Institute of Hydrodynamics, Novosibirsk, 630090, Russia

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Asymptotic and numerical analyses of unsteady hydroelastic behaviour of a floating plate due to given external loads are presented. The main parameters are the plate length and duration of the external loads. For very long plates (VLFS) the problem is decoupled and its approximate solution is given by the method of matched asymptotic expansions. For a short duration of the external loads and small length of the plate (impact onto a floating plate) the problem is coupled, but gravity effects can be neglected in determining the maximum of both the plate deflection and bending stresses in the plate. In this case, the problem is solved numerically by the method of normal modes. If the plate is short but the load duration is moderate, the rigid-body motion of the plate and its elastic vibrations can be approximately separated. In the general case, it is suggested that the coupled problem can be treated numerically by the method of normal modes. In order to construct an appropriate numerical algorithm, ideas inspired by the asymptotic analysis are used.

1. INTRODUCTION

THE PLANE UNSTEADY PROBLEM of the hydroelastic behaviour of a plate floating on a liquid surface is considered within the framework of linear theory. The plate deflection is due to a given external load. Initially, the liquid is at rest and occupies a lower half-plane ($y' < 0$). The part of the liquid boundary $-L < x' < L$, $y' = 0$ corresponds to the floating elastic plate (see Figure 1), and the parts $x' < -L$ and $x' > L$, $y' = 0$ correspond to the free surface, where the pressure is zero at all times. The plate centre is taken as the origin of the Cartesian coordinate system $x'Oy'$ (dimensional variables are denoted by a prime). At some instant of time, taken as the initial state ($t' = 0$), the prescribed external load $Qq(x'/Ll, t'/T_q)$ starts to force the plate to move, where l is the ratio of the external load area to the plate length, T_q is the characteristic time of the load duration, Q is the magnitude of the total load and q is the nondimensional function. The shape of the plate is changing owing to both the transient external loads and the plate interaction with the liquid.

We shall determine both the elastic deflections and the bending stress distribution in the plate under the following assumptions: (i) the dynamics of the plate deflection is governed by the Euler beam equation; (ii) the end points of the plate are free of stresses; (iii) both the plate thickness h and the plate draft at rest, d, are much smaller than the plate length, $2L$; (iv) the external loads are symmetrical with respect to the plate centre; (v) the liquid is ideal and incompressible; (vi) the liquid flow is plane, potential and symmetrical with respect to the y'-axis; and (vii) the liquid motion is described within the framework of linear theory.

The goal of this paper is to distinguish limiting cases, which allow further simplifications of the problem and its approximate solutions. It is believed that asymptotic methods will be helpful in developing an adequate numerical algorithm, and dealing with the hydroelastic

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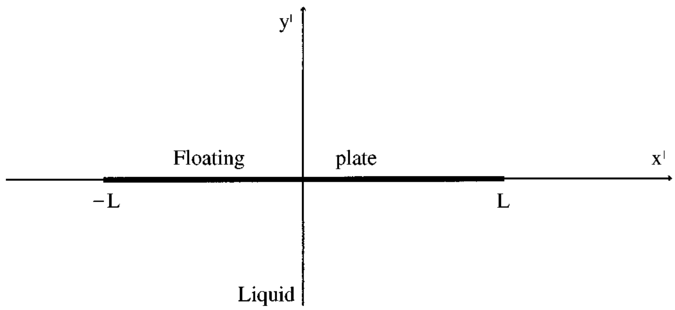

Figure 1. Unsteady behaviour of a floating elastic plate. Initially, the liquid is at rest and occupies a lower half-plane $y' < 0$, the plate corresponds to the interval $-L < x' < L$.

problems in cases for which direct numerical calculations are time-consuming, and do not clarify the mechanics of the phenomena.

Motivation for the present study comes from the problem of a spacecraft being launched from an artificial floating island. The spacecraft is placed at the centre of the floating platform, the initial deflection of which is not taken into account. External loads on the floating platform are due to the gas jet during the launching. The asymptotic analysis presented may be of interest for study of landing and take-off of airplanes from a floating runway, as well as for the problem of huge mass impact onto a floating platform. The plane hydroelastic problem, which is under consideration here, provides an outline of the method and demonstrates its peculiarities. The approximate models derived can be readily generalized to three-dimensional problems of hydroelasticity of floating plates of an arbitrary shape.

The traditional approach for treating hydroelastic problems is to divide them into the following two parts: a hydrodynamic problem for the liquid flow and an elastic problem for the plate deflection. In general, these parts are connected to each other: the liquid flow can be determined if the plate deflection is known, and the elastic deflection of the plate can be found only if the hydrodynamic loads are prescribed. This means that both parts of the original problem have to be considered at the same time. In order to solve the coupled problem of hydroelasticity, the general solution of the hydrodynamic problem and that of the elastic problem are employed. Combining these solutions, we arrive at an integro-differential equation with respect to the plate deflection.

It should be noted that the integrals in this integro-differential equation are only over the plate region if the effects of gravity are neglected. In this case the unsteady problem can easily be solved if the response matrix for plate oscillations in an unbounded liquid is known. This implies that, without gravity effects, the unsteady hydroelastic problem can be divided into two parts: (a) a steady problem to evaluate the response matrix; (b) an unsteady problem to determine the principal coordinates of the plate vibration modes. The last problem can be reduced to the initial-value problem for an infinite system of ordinary differential equations with constant coefficients.

When the effects of gravity are taken into account, the problem of a floating elastic plate becomes more complicated. In this problem the integrals in the integro-differential equation are not over the plate only but in time also, which is due to memory effects. In this case iterative procedures based on decoupling the original unsteady problem into the hydrodynamic and elastic parts may be misleading, because they do not give us any clear idea about how to treat the original problem numerically. The main reason for this is connected with the fact that the hydrodynamic loads on an elastic plate are strongly dependent on the plate acceleration, and the corresponding terms give important contributions to the equation for the plate deflection. This point is not of major significance if the external loads on the plate are periodic in time and we search for the time-periodic deflection of the plate, but it is crucial for general unsteady problems. Nevertheless, even for the problem of the time-periodic deflection of a floating plate, some difficulties in numerical simulations based on traditional approaches may arise in the case of high-frequency excitations.

In its general form, the unsteady problem of hydroelasticity is very complicated, which is why simple asymptotic solutions valid in special cases may be helpful in clarifying the contributions of different effects (gravity, structural inertia, restoring forces and hydrodynamic loads). Three cases may be distinguished: (i) for very long plates which are extremely flexible, structural inertia forces can be neglected, and the restoring forces are of importance only close to the plate periphery and near locations where concentrated loads are applied; (ii) for external loads of short duration (impact loads) and plates of moderate length, the extreme deflections and stresses can be estimated without taking gravity into account, in the same way as for the problem of wave impact onto an elastic plate; (iii) if the duration of external loads is comparable with the time scale $T_g = \sqrt{L/g}$ for the gravity effects, and the plate is not very long, then the hydrodynamic loads are approximately independent of the elastic deflection of the plate and are determined in the leading order by the plate-rigid-body motion.

2. FORMULATION OF THE PROBLEM

The plane linear problem of an elastic plate floating on the surface of an ideal and incompressible liquid is considered in nondimensional variables. Unsteady motion of the plate is due to given external loads. The plate deflection $w(x, t)$ is governed by the Euler beam equation, and the irrotational liquid flow is described by the velocity potential $\varphi(x, y, t)$. The nondimensional variables and unknown functions are introduced as follows:

$$x' = Lx, \quad y' = Ly, \qquad t' = Tt, \qquad s' = S_{sc}\, s,$$

$$w' = Ww, \qquad \varphi' = \Phi_{sc}\varphi, \qquad p' = P_{sc}p,$$

where $s(t)$ is the rigid displacement of the plate and $p(x, y, t)$ is the hydrodynamic pressure. The scales T, S_{sc}, W, Φ_{sc} and P_{sc}, which are dependent on both the parameters of the plate and those of the external loads, will be specified later. The coupled problem of the liquid flow and the plate motion has the form

$$\alpha_E \frac{\partial^2 w}{\partial t^2} - \alpha_R \frac{\partial^2 s}{\partial t^2} + \beta_0 \frac{\partial^4 w}{\partial x^4} = vp(x, 0, t) - q(x/l, \omega t) \qquad (t > 0, |x| < 1), \tag{1}$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial^3 w}{\partial x^3} = 0 \qquad (t \geq 0, x = \pm 1), \tag{2}$$

$$w(x, 0) = 0, \qquad \frac{\partial w}{\partial t}(x, 0) = 0 \qquad (|x| \leq 1), \tag{3}$$

$$\Delta\varphi = 0 \qquad (y < 0), \tag{4}$$

$$\frac{\partial^2 \varphi}{\partial t^2} + \gamma \frac{\partial \varphi}{\partial y} = 0 \qquad (y = 0, |x| > 1), \tag{5}$$

$$\frac{\partial \varphi}{\partial y} = \kappa_E \frac{\partial w}{\partial t} - \kappa_R \frac{\partial s}{\partial t} \qquad (y = 0, |x| < 1), \tag{6}$$

$$\varphi = 0, \qquad \frac{\partial \varphi}{\partial t} = 0 \qquad (t = 0, y = 0, |x| > 1), \tag{7}$$

$$-p(x, 0, t) = \lambda \frac{\partial \varphi}{\partial t}(x, 0, t) + \mu_E w(x, t) + \mu_R s(t) \qquad (|x| < 1, t \geq 0), \tag{8}$$

$$\int_{-1}^{1} w(x, t)\,\mathrm{d}x = 0. \tag{9}$$

Equation (8) is the linearized Cauchy–Lagrange integral and equation (9) is the orthogonality condition to separate rigid and elastic motions of the plate. Once the beam deflection $w(x, t)$ has been found, the bending stresses in the beam $\sigma(x, t)$ are given in the nondimensional variables by the formula $\sigma(x, t) = w_{xx}(x, t)$, the stress scale being $EWh/(2L^2)$. The initial-value problem (1)–(9) contains 11 "free" nondimensional parameters:

$$\alpha_E = \frac{\rho W d}{T^2 Q}, \qquad \alpha_R = \frac{\rho S_{sc} d}{T^2 Q}, \qquad \beta_0 = \frac{EJW}{L^4 Q}, \qquad \omega = \frac{T}{T_q},$$

$$\nu = \frac{P_{sc}}{Q}, \qquad \kappa_E = \frac{LW}{T\Phi_{sc}}, \qquad \kappa_R = \frac{LS_{sc}}{T\Phi_{sc}}, \qquad \gamma = \frac{gT^2}{L},$$

$$\lambda = \frac{\rho \Phi_{sc}}{PT}, \qquad \mu_E = \frac{\rho g W}{P_{sc}}, \qquad \mu_R = \frac{\rho g S_{sc}}{P_{sc}},$$

which can be specified with an appropriate choice of the scales, and three "fixed" nondimensional parameters,

$$\frac{d}{L}, \qquad \beta = \frac{EJ}{\rho g L^4}, \qquad l,$$

which characterize the plate properties and the load distribution. Only six of the "free" parameters, ν, γ, κ_E, κ_R, λ and ω, are independent and other parameters can be expressed with their help:

$$\alpha_E = \frac{d}{L} \nu\lambda\kappa_E, \qquad \alpha_R = \frac{d}{L} \nu\lambda\kappa_R, \tag{10}$$

$$\beta_0 = \beta\nu\mu_E, \qquad \mu_E = \lambda\gamma\kappa_E, \qquad \mu_R = \lambda\gamma\kappa_R.$$

Here g is the acceleration due to gravity, E is the elasticity modulus, J is the second moment of inertia per unit width of the beam cross-section; $J = h^3/12$ for a beam of uniform thickness h. The condition of the plate equilibrium at the initial moment provides that the plate mass per unit plate area is equal to ρd, where d is the draft of the floating plate. The parameter β does not depend on external loads and is usually small for large floating structures.

The scales have to be specified in such a way that all coefficients in (1)–(9) are equal to or less than unity and that the resulting equations provide a nontrivial problem even when the coefficients which are less than unity are replaced by zero. According to this rule and taking equation (10) into account, we obtain $\lambda = 1$ [if not, equation (8) is trivial as λ is replaced by zero], $v = 1$ [if not, equation (1) can be reduced to a trivial one] and $\kappa_R = 1$ [if not, the problem is trivial for $\kappa_E = 0$]. Finally, $P_{sc} = Q$, $\Phi_{sc} = QT/\rho$, $S_{sc} = QT^2/\rho L$ but the time scale T and the deflection scale W are still not specified. Moreover, $\alpha_R = d/L$ and α_E is less than the ratio d/L, which is small corresponding to the main assumptions of the paper. Therefore, we can neglect the structure inertia terms in the further analysis.

It is convenient to denote the ratio W/S_{sc} by κ. Equations (1), (6) and (8), forms of which are dependent on the values of the parameters β γ and κ, can be rewritten as

$$\beta\gamma\kappa \frac{\partial^4 w}{\partial x^4} = p(x, 0, t) - q(x/l, \omega t), \tag{11}$$

$$p(x, 0, t) = -\frac{\partial \varphi}{\partial t}(x, 0, t) - \gamma\kappa w(x, t) + \gamma s(t), \tag{12}$$

$$\frac{\partial \varphi}{\partial y}(x, 0, t) = \kappa \frac{\partial w}{\partial t} - \frac{\partial s}{\partial t} \qquad (|x| < 1), \tag{13}$$

respectively.

We consider in Sections 3–5 only the case where the external load distribution along the plate is smooth, $l = 1$. In this case the scales T and W are dependent on the orders of the parameters β and ω, and have to be specified in such a way that it would be possible to evaluate maxima of both deflection and stress in the plate and to determine the high-order approximations of the solution. This will be done below by the methods of asymptotic analysis.

3. ASYMPTOTIC ANALYSIS

The scales T and W have to be defined in such a way that $\kappa \leq 1$, $\gamma \leq 1$ and $\beta\gamma\kappa \leq 1$. Moreover, the linear theory of floating plate hydroelasticity can be justified if and only if $Q \ll \rho L^2/T^2$.

Three time scales, T_q, T_g and $T_w = \sqrt{\rho L^5/EJ}$, which are the scales for different effects, can be distinguished in the problem under consideration. The first scale is associated with the duration of the external loads, the second with the gravity effects, and the third with the largest period of vibration of the beam floating on the surface of the liquid in the absence of gravity. It is seen that $T_g = \beta^{1/2} T_w$, which implies that the importance of the gravity effects strongly depends on the order of the parameter β. A beam is referred to as long if $\beta \ll 1$, and short if $\beta \gg 1$. If $\beta = O(1)$, the beam length is referred to as moderate. For example, for a floating airport having bending rigidity $EJ \approx 1{\cdot}8 \times 10^{11}$ N m (Kashiwagi 1998) the parameter β is unity for the plate length $2L$ of about 150 m and is approximately $0{\cdot}5 \times 10^{-6}$ for $2L = 5000$ m. For the model experiments conducted at the Ship Research Institute (Yago & Endo 1996) with $EJ = 1{\cdot}752 \times 10^4$ N m the parameter β is unity for a plate length of about 2 m. In the actual experimental conditions, $2L = 9{\cdot}75$ m, and we find that $\beta = 0{\cdot}0032$, which is much less than unity.

For long plates, the characteristic period of the plate vibration, T_w, is much greater than the scale, T_g, for the gravity effects. This is why the latter give the main contribution and balance the external loads. We take T_g as the time scale and W equal to S_{sc}, which yields $\kappa = 1$, $\gamma = 1$ and $\beta\gamma\kappa \ll 1$.

In the case of short plates, $\beta \gg 1$, and small duration of external loads, $T_q/T_g \ll 1$, the plate starts to vibrate due to the impact, with the vibration period being of the order T_w. The amplitude of the vibrations is weakly dependent on the gravity effects at this initial stage, $t/T_w = O(1)$, and slowly decays with increasing time. This implies that gravity effects can be neglected in the leading order in evaluation of the maximum amplitude of the plate deflection. In this case, the quantity T_w is taken as the time scale and $W = S_{sc}$. We obtain $\beta\gamma\kappa = 1$, $\gamma = \beta^{-1}$ and $\kappa = 1$. The separation of the rigid and elastic motions of the plate is not necessary, and the right-hand side of equation (13) can be replaced by $\partial w/\partial t$, where now $w(x, t)$ is the local displacement of plate elements.

However, if $T_q/T_g = O(1)$, the scales T and W for short plates, $\beta \gg 1$, are different from those in the impact case. Now we have to take T_g as the time scale, which gives $\gamma = 1$, and $\kappa = \beta^{-1}$. In this case the plate can be considered as rigid in the leading order as $\beta^{-1} \to 0$ and its elastic deflections can be determined from the beam equation (11), where the hydrodynamic pressure $p(x, 0, t)$ is given approximately by the solution of the boundary-value problem (4)–(8) without account for the plate flexibility, $w(x, t) = 0$.

In the case of concentrated loads, $l \ll 1$, the asymptotic classification is the same as for smoothly distributed loads but "inner" solutions are required close to the region of the load action. The case of periodically distributed external loads along the plate, which is the main interest in the problem of wave action on a floating plate, is not considered here.

4. HYDROELASTIC BEHAVIOUR OF LONG PLATES

For long plates ($\kappa = 1$, $\gamma = 1$ and $\beta \ll 1$), equation (11) predicts that the presence of the floating plate can be disregarded in the leading order as $\beta \to 0$. The external loads may be considered as being applied directly to the liquid free surface, and the plate follows the free surface shape. Within the framework of this approach the boundary conditions at the beam edges (2) are not satisfied, which implies that "inner" asymptotic solutions close to the beam edges have to be constructed. In the following the rigid and elastic motions of the plate are not separated and $w(x, t) - s(t)$ is replaced by $w(x, t)$ in equations (12) and (13). Correspondingly, the orthogonality condition (9) is omitted.

The boundary-value problem (2)–(5), (7) and (11)–(13) is rewritten now as

$$\Delta\varphi = 0 \qquad (y < 0), \tag{14}$$

$$\frac{\partial^2\varphi}{\partial t^2} + \frac{\partial\varphi}{\partial y} = -\frac{\partial p}{\partial t}(x, 0, t) \qquad (y = 0), \tag{15}$$

$$p(x, 0, t) = 0 \qquad (y = 0, |x| > 1), \tag{16}$$

$$p(x, 0, t) = q(x, \omega t) + \beta\frac{\partial^4 w}{\partial x^4} \qquad (y = 0, |x| < 1), \tag{17}$$

$$\frac{\partial w}{\partial t}(x, t) = \frac{\partial\varphi}{\partial y}(x, 0, t) \qquad (y = 0, |x| < 1), \tag{18}$$

$$\varphi = 0, \qquad \frac{\partial\varphi}{\partial t} = 0 \qquad (t = 0, y = 0, |x| > 1), \tag{19}$$

$$w(x, 0) = 0, \qquad \frac{\partial w}{\partial t}(x, 0) = 0 \qquad (|x| \le 1). \tag{20}$$

It should be noted that the boundary conditions (2) are not taken into account in problem (14)–(20), the solution of which will be referred to as the "outer" solution. The "outer" asymptotic solution is sought in the form

$$\varphi(x, y, t) = \varphi^{(0)}(x, y, t) + \beta\varphi^{(1)}(x, y, t) + \cdots, \tag{21}$$

$$w(x, t) = w^{(0)}(x, t) + \beta w^{(1)}(x, t) + \cdots. \tag{22}$$

Substituting equations (21) and (22) into equations (14)–(20), we obtain in the leading order that $\varphi^{(0)}(x, y, t)$ satisfies equations (14), (19) and the boundary condition on the liquid surface, $y = 0$,

$$\frac{\partial^2 \varphi^{(0)}}{\partial t^2} + \frac{\partial \varphi^{(0)}}{\partial y} = -\frac{\partial q}{\partial t}(x, \omega t)\mathrm{H}(1 - x^2), \tag{23}$$

where $\mathrm{H}(x)$ is the Heaviside function. The beam deflection $w^{(0)}(x, t)$ is governed by the following initial problem:

$$\frac{\partial w^{(0)}}{\partial t} = \frac{\partial \varphi^{(0)}}{\partial y}(x, 0, t) \qquad (|x| < 1, t \geq 0), \tag{24}$$

$$w^{(0)}(x, 0) = 0 \qquad (|x| < 1). \tag{25}$$

Correspondingly, $\varphi^{(1)}(x, y, t)$ satisfies equations (14) and (19), the boundary condition on the line $y = 0$,

$$\frac{\partial^2 \varphi^{(1)}}{\partial t^2} + \frac{\partial \varphi^{(1)}}{\partial y} = -\frac{\partial^5 w^{(0)}}{\partial x^4 \partial t}(x, t)\mathrm{H}(1 - x^2), \tag{26}$$

and the matching conditions of the "outer" solution with the "inner" ones, which will be derived below. The function $w^{(1)}(x, t)$ is governed by the following equations:

$$\frac{\partial w^{(1)}}{\partial t} = \frac{\partial \varphi^{(1)}}{\partial y}(x, 0, t) \qquad (|x| < 1, t \geq 0), \tag{27}$$

$$w^{(1)}(x, 0) = 0 \qquad (|x| < 1). \tag{28}$$

For smooth external loads the high-order approximations of the solution can readily be computed.

The "outer" solution does not satisfy the edge conditions (2) and the "inner" solutions, which describe the fine structure of the flow close to the beam edges, have to be obtained. In a region near the right-hand side edge ($x = 1$, $y = 0$), the size of which is dependent on the parameter β and decreases as $\beta \to 0$, the "inner" solution is assumed to be in the form

$$\begin{gathered} x = 1 + \beta^n \xi, \qquad y = \beta^n \eta, \\ w(x, t) = w_{\text{outer}}(x, t) + \beta^k w_i(\xi, t), \\ \varphi(x, y, t) = \varphi_{\text{outer}}(x, y, t) + \beta^m \varphi_i(\xi, \eta, t), \\ p(x, y, t) = p_{\text{outer}}(x, y, t) + \beta^j p_i(\xi, \eta, t), \end{gathered} \tag{29}$$

where $w_{\text{outer}}(x, t)$ and $\varphi_{\text{outer}}(x, y, t)$ are given by equations (22) and (21), respectively. The quantity n determines the size of the "inner" region, where solution (29) holds. The constants k, m, j and n have to be found together with the 'inner' solution, to provide a nontrivial description of the flow near the edge.

Substituting equation (29) into equations (14)–(20) and the edge conditions (2), and comparing the orders of coefficients in these equations as $\beta \to 0$, we find $n = \frac{1}{4}$, $k = \frac{1}{2}$, $j = \frac{1}{2}$ and $m = \frac{3}{4}$. The hydroelastic behaviour of the beam in the "inner" region is governed by the equations

$$\frac{\partial^2 \varphi_i}{\partial \xi^2} + \frac{\partial^2 \varphi_i}{\partial \eta^2} = 0 \qquad (\eta < 0), \tag{30}$$

$$\frac{\partial^4 w_i}{\partial \xi^4} + w_i = -\beta^{1/4} \frac{\partial \varphi_i}{\partial t} \qquad (\xi < 0), \tag{31}$$

$$\frac{\partial w_i}{\partial t} = \frac{\partial \varphi_i}{\partial \eta} \qquad (\eta = 0,\ \xi < 0), \tag{32}$$

$$\frac{\partial \varphi_i}{\partial \eta} = -\beta^{1/4} \frac{\partial^2 \varphi_i}{\partial t^2} \qquad (\eta = 0,\ \xi > 0), \tag{33}$$

$$\frac{\partial^2 w_i}{\partial \xi^2}(0, t) = -\frac{\partial^2 w_{\text{outer}}}{\partial x^2}(1, t), \tag{34}$$

$$\frac{\partial^3 w_i}{\partial \xi^3}(0, t) = -\beta^{1/4} \frac{\partial^3 w_{\text{outer}}}{\partial x^3}(1, t), \tag{35}$$

$$w_i(\xi, 0) = 0, \tag{36}$$

$$w_i(\xi, t) \to 0 \qquad (\xi \to -\infty), \tag{37}$$

where the latter condition follows from the condition of matching solutions in the "outer" and "inner" regions. The bending stresses $\sigma_i(\xi, t)$ are given in the "inner" region by the formula

$$\sigma_i(\xi, t) = \frac{\partial^2 w_i}{\partial \xi^2} \qquad (\xi < 0) \tag{38}$$

with the same scale $EWh/(2L^2)$, where $W = Q/\rho g$, as that for the "outer" solution.

We denote

$$\frac{\partial^2 w_{\text{outer}}}{\partial x^2}(1, t) = \frac{\partial^2 w^{(0)}}{\partial x^2}(1, t) + O(\beta) =: a(t) + O(\beta),$$

$$\frac{\partial^3 w_{\text{outer}}}{\partial x^3}(1, t) = \frac{\partial^3 w^{(0)}}{\partial x^3}(1, t) + O(\beta) =: b(t) + O(\beta),$$

and represent the "inner" solution by the following asymptotic expansions:

$$w_i(\xi, t) = w_i^{(0)}(\xi, t) + \beta^{1/4} w_i^{(1)}(\xi, t) + \cdots,$$

$$\varphi_i(\xi, \eta, t) = \varphi_i^{(0)}(\xi, \eta, t) + \beta^{1/4} \varphi_i^{(1)}(\xi, \eta, t) + \cdots. \tag{39}$$

The substitution of equation (39) into equations (30)–(37) gives in the leading order the boundary-value problem for the function $w_i^{(0)}(\xi, t)$:

$$\frac{\partial^4 w_i^{(0)}}{\partial \xi^4} + w_i^{(0)} = 0 \qquad (\xi < 0),$$

$$\frac{\partial^2 w_i^{(0)}}{\partial \xi^2} = -a(t), \qquad \frac{\partial^3 w_i^{(0)}}{\partial \xi^3} = 0 \qquad (\xi = 0), \tag{40}$$

$$w_i^{(0)}(\xi, t) \to 0 \qquad (\xi \to -\infty),$$

and the boundary-value problem for the velocity potential $\varphi_i^{(0)}(\xi, \eta, t)$:

$$\Delta\varphi_i^{(0)} = 0 \qquad (\eta < 0),$$
$$\frac{\partial\varphi_i^{(0)}}{\partial\eta} = \frac{\partial w_i^{(0)}}{\partial t} \qquad (\eta = 0,\ \xi < 0), \tag{41}$$
$$\frac{\partial\varphi_i^{(0)}}{\partial\eta} = 0 \qquad (\eta = 0,\ \xi > 0),$$
$$\varphi_i^{(0)} \to 0 \qquad (\xi^2 + \eta^2 \to \infty).$$

It can be shown that

$$\int_{-\infty}^{\infty} (\partial\varphi_i^{(0)}/\partial\eta)(\xi, 0, t)\,\mathrm{d}\xi = 0,$$

which implies that the solution of problem (41) exists and is unique. The solution of the boundary-value problem (40) is

$$w_i^{(0)}(\xi, t) = -\,a(t)\mathrm{e}^{\xi/\sqrt{2}}[\cos(\xi/\sqrt{2}) + \sin(\xi/\sqrt{2})], \tag{42}$$
$$\sigma_i^{(0)}(\xi, t) = -\,a(t)e^{\xi/\sqrt{2}}[\cos(\xi/\sqrt{2}) - \sin(\xi/\sqrt{2})].$$

Both the plate deflection and the bending stresses oscillate close to the beam edge with wavelength equal to $2\pi(4EJ/\rho g)^{1/4}$ in the dimensional variables, which does not depend on the plate length. The amplitude of these oscillations decays exponentially with the distance from the edge. The maximum deviation of the beam shape from the shape of the liquid surface, $\Delta w(t)$, takes place at the beam edge and is given in the leading order as

$$\Delta w(t) = -\,a(t)\beta^{1/4} + O(\beta^{1/2}).$$

If $\Delta w(t) > d$, which implies $a(t) < -\mathrm{d}\beta^{-1/4}$, the beam edge exits the water and the phenomenon of slamming may be observed.

Formulae (42) indicate that the deviation of the solution from the "outer" one is localized close to the beam edge and

$$\int_{-\infty}^{0} w_i^{(0)}(\xi, t)\,\mathrm{d}\xi = 0, \qquad \int_{-\infty}^{0} w_i^{(0)}(\xi, t)\xi\,\mathrm{d}\xi = -\,a(t).$$

These integral equalities provide that the liquid flow far from the beam edge, $\xi^2 + \eta^2 \gg 1$, can be approximately described by the solution of problem (41) with the boundary condition on the liquid surface being replaced by a simpler one

$$\frac{\partial\varphi_i^{(0)}}{\partial\eta} = \frac{\mathrm{d}a}{\mathrm{d}t}\,\delta'(\xi) \qquad (\eta = 0),$$

where $\delta'(\xi)$ is the first derivative of the Dirac delta-function. We obtain

$$\varphi_i^{(0)}(\xi, \eta, t) \sim -\frac{1}{\pi}\frac{\mathrm{d}a}{\mathrm{d}t}\frac{\xi}{\xi^2 + \eta^2} \qquad (\xi^2 + \eta^2 \to \infty). \tag{43}$$

Asymptotic formula (43) together with expansions (21) and (39) show that the first-order "outer" velocity potential $\varphi^{(1)}(x, y, t)$ is singular at the beam edges, where

$$\varphi^{(1)}(x, y, t) \sim -\frac{1}{\pi}\frac{\mathrm{d}a}{\mathrm{d}t}\frac{|x|-1}{(|x|-1)^2+y^2} \tag{44}$$

as $y^2 + (|x| - 1)^2 \to 0$. The matching condition (44) can be combined with the boundary condition (26) to give

$$\frac{\partial^2\varphi^{(1)}}{\partial t^2} + \frac{\partial\varphi^{(1)}}{\partial y} = -\frac{\partial^5 w^{(0)}}{\partial x^4 \partial t}(x, t)\mathrm{H}(1-x^2) + \frac{\mathrm{d}a}{\mathrm{d}t}[\delta'(x-1) + \delta'(x+1)] \qquad (y = 0).$$

The asymptotic analysis demonstrates that the main part of a long floating beam follows the shape of the liquid free surface, but the beam edges do not. In order to determine the high-order approximations of the beam deflection, generalized functions have to be used, which makes the construction of analytical solutions quite complicated.

The asymptotic procedure described can be employed after simple modifications for analysis of the hydroelastic behaviour of an elastic plate in incident periodic waves. The plate deflection is assumed to be periodic in time with the period of its vibrations being the same as that of the plane incident wave. We consider the case where the presence of the floating plate weakly effects the flow. This is possible for very flexible plates and/or for very long incident waves. We assume that, in the leading order, the plate follows the shape of the incident wave, which gives the plate deflection far from its edges. This simple solution is considered as the "outer" solution valid for the main part of the plate. Close to the edges the "inner" solutions similar to equation (42) are used to correct the "outer" solution. Combination of the "outer" and "inner" solutions provides the uniformly valid approximation of the plate deflection. It is found that this approximation is formally valid for incident waves, the length of which is much greater than $2\pi\beta^{1/4}L$. The approximation is referred to as the "long-wave approximation". Nondimensional amplitudes of the bending stresses given by the long-wave approximation are compared with the experimental results by Wu *et al.* (1995) in Figure 2 for a wave period of 1·429 s. The notation is the same as in the paper by Wu *et al.* (1995). Three wave periods were considered in the experiments: 0·7 s, 1·429 s (wavelength is 3·1 m) and 2·875 s (wavelength is 8·6 m). The plate is 10 m long and 38 mm thick, and the water is 1·1 m deep. In Figure 2 experimental results are shown with circles and the solution given by the long-wave approximation with the broken line. Direct numerical simulations of the hydroelastic behaviour of the plate in incident waves (plane case) were also performed. The numerical results are shown with the solid line. The numerical method used (Korobkin 1998) is similar to that by Wu *et al.* (1995) but now the hydrodynamic coefficients are calculated analytically. It was found that the approximate analytical solution slightly underpredicts the stress amplitude for the wavelength of 8·6 m, and does not correspond to the measured values for the wavelength of 3·1 m. It should be noted that the results of direct numerical computations (Korobkin 1998) are identical to those reported by Sturova (1998), who used the domain decomposition method, and are slightly different from the numerical results by Wu *et al.* (1995). On the other hand, numerical predictions by Wu *et al.* (1995), Sturova (1998) and Korobkin (1998) for short waves with a wave period of 0·7 s neither correspond to each other nor to the measured values of bending stresses.

The approach described in this section can be applied to the three-dimensional problem of hydroelastic behaviour of a floating plate of an arbitrary shape due to transient external loads or incident waves. If the plate shape is smooth, the approach can be applied directly. If the plate has corner points, the "inner" three-dimensional solutions have to be obtained close to these points. This problem is not considered here.

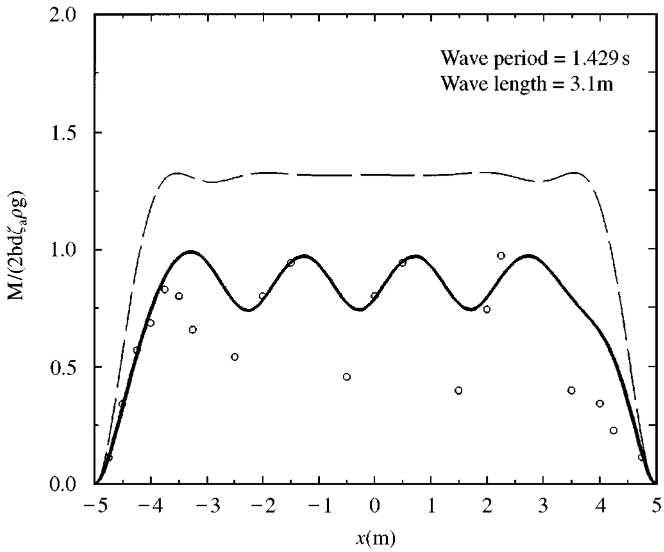

Figure 2. Nondimensional amplitude of bending stresses for the elastic plate in waves: ——, Numerical analysis; – – –, long-wave approximation; ○, experimental values.

5. HYDROELASTIC BEHAVIOUR OF SHORT PLATES

In the case of a short plate, $\beta \gg 1$, and a duration of the external loads T_q comparable with T_g, $\omega = 1$, elastic deflection of the plate is small compared to the plate rigid displacement, $W = S_{sc}/\beta$. The plate can be considered as rigid in the leading order as $\beta \to \infty$ and its structural mass can be neglected compared to the added mass of the plate.

According to the method of Cummins (1962), the velocity potential $\varphi(x, y, t)$ of the flow caused by a plate undergoing heave motion is given by

$$\varphi(x, y, t) = -\dot{s}\psi(x, y) + \int_0^t \Phi(x, y, t-\tau)\dot{s}(\tau)\,\mathrm{d}\tau, \tag{45}$$

where $\dot{s} = (\mathrm{d}s/\mathrm{d}t)(t)$; $\psi(x, y)$ satisfies the Laplace equation in the lower half-plane, $y < 0$, it is equal to zero on the free surface, $y = 0$, $|x| > 1$, and its normal derivative $\mathrm{d}\psi/\mathrm{d}y$ is unity on the plate, $y = 0$, $|x| < 1$. The function $\Phi(x, y, t)$ satisfies the following equations:

$$\begin{aligned} &\Delta\Phi = 0 \qquad (y < 0),\\ &\Phi_{tt} + \Phi_y = 0 \qquad (y = 0, |x| > 1),\\ &\Phi_y = 0 \qquad (y = 0, |x| < 1),\\ &\Phi(x, y, 0) = 0,\\ &\Phi_t(x, 0, 0) = \partial\psi/\partial y(x, 0) \qquad (|x| > 1). \end{aligned} \tag{46}$$

It is well known (Wagner 1932) that $\psi(x, 0) = \sqrt{1 - x^2}$, where $|x| < 1$, and $(\partial\psi/\partial y)(x, 0) = 1 - |x|/\sqrt{x^2 - 1}$, where $|x| > 1$.

Integration of equation (11) along the plate, taking into account equations (12) and (45), where $\gamma = 1$, $\kappa = \beta^{-1}$ and $\beta \to \infty$, yields the equation for the amplitude of the plate heave motion

$$\ddot{s} + \frac{4}{\pi} s = \int_0^t \mu(t-\tau)\dot{s}(\tau)\,\mathrm{d}\tau + \frac{2}{\pi}\int_{-1}^{1} q(x,t)\,\mathrm{d}x, \tag{47}$$

$$\mu(t) = \frac{2}{\pi}\frac{\mathrm{d}}{\mathrm{d}t}\int_{-1}^{1} \Phi(x,0,t)\,\mathrm{d}x, \qquad \mu(0) = \frac{4}{\pi}.$$

The initial conditions for equation (47) are

$$s(0) = 0, \qquad \dot{s}(0) = 0. \tag{48}$$

Once the initial-value problem (47) and (48) has been solved, the beam deflection is governed in the leading order by the following boundary-value problem:

$$\frac{\partial^4 w}{\partial x^4} = \ddot{s}(t)\sqrt{1-x^2} + s(t) - q(x,t) - \int_0^t \Phi(x,0,t-\tau)\dot{s}(\tau)\,\mathrm{d}\tau \qquad (|x|<1), \tag{49}$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial^3 w}{\partial x^3} = 0 \qquad (x = \pm 1), \tag{50}$$

where $s(t)$, $\dot{s}(t)$ and $\ddot{s}(t)$ are given by equations (47) and (48). If the function $\mu(t)$ is known in advance, it is not difficult to find numerically the solutions of equations (47) and (49). In order to determine $\mu(t)$, the initial-value problem (46) has to be solved. We find that, for $|x| < 1$,

$$\Phi_t(x,0,0) = 1 - \frac{1}{\pi\sqrt{1-x^2}}\left(2 + x\ln\frac{1-x}{1+x}\right),$$

which indicates that the initial asymptotics of the function $\Phi(x, y, t)$ are not uniformly valid and the "inner" solutions have to be constructed close to the plate edges. Therefore, special care is required in computing the distribution of the velocity potential $\Phi(x, 0, t)$ along the plate, especially for small times. The plane unsteady problem of heave motion of a horizontal rigid plate is beyond the scope of the present analysis and is not considered here.

The short-plate approximation, which is for $\beta \gg 1$, can be used not only in the case of external loads of large duration considered above, but also in the case of loads of very short duration (impact loads), $T_q/T_g \ll 1$. In the latter case the method of matched asymptotic expansions with the ratio T_q/T_g as a small parameter of the problem indicates that an initial stage of the interaction between the plate and the liquid can be distinguished (see Section 6). During this stage, the plate flexibility is of major importance, but gravity effects are not well pronounced yet, and can be approximately neglected in the leading order as $T_q/T_g \to 0$. Without gravity effects being taken into account, the velocity of the rigid displacement of the plate approaches, in dimensional variables, the constant value

$$V_* = \left(2\rho d L^2 + \frac{\pi}{2}\rho L^2\right)^{-1} Q \int_0^{T_q}\int_{-L}^{L} q(x'/Ll, t'/T_q)\,\mathrm{d}x'\,\mathrm{d}t'$$

at the end of the initial stage. The last formula follows from equation (1) under the assumption that at the end of the impact stage the liquid flow due to elastic vibrations of the plate can be neglected compared to that caused by the rigid motion of the plate. The normal mode method is employed in Section 6 to analyse the plate motion during the initial stage. During the next stage, where gravity effects are of major significance but elastic effects have died out already, the short-plate approximation can be used. Rigid motion of the plate is described by equation (47), the initial conditions for which, in accordance with the method of matched asymptotic expansions, are

$$s(0) = 0, \qquad \dot{s}(0) = -1,$$

with the product $V_* T_g$ chosen as the displacement scale.

It is seen that rigid motion of a short plate can approximately be determined independently of the hydroelastic behaviour of the plate during the initial stage. On the other hand, the absolute maximum of bending stresses occurs in the initial stage, which is a reason for analysing this stage in detail.

6. IMPACT ONTO A FLOATING SHORT PLATE

In the case under consideration in this section the plate is short, $\beta \gg 1$, but the duration of the external loads T_q is much less than T_g. The plate vibration period T_w is taken as the time scale of the process. The parameter γ is equal to β^{-1} and is much less than unity, which implies that the gravity effects can be disregarded at the initial stage, where the bending stresses in the plate peak. Within this simplified formulation, the memory effects are not taken into account and the method of normal modes can be used. The velocity potential $\varphi(x, y, t)$ satisfies the Laplace equation in the lower half-plane, $y < 0$, is equal to zero on the free surface, $y = 0$, $|x| > 1$, in the leading order as $\beta^{-1} \to 0$, and satisfies the kinematic condition $\partial\varphi/\partial y = \partial w/\partial t$ on the plate, $y = 0$, $|x| < 1$. It should be noted that we do not separate now elastic and rigid motions of the plate.

The velocity potential on the plate and the beam deflection, $|x| < 1$, are sought in the forms

$$\varphi(x, 0, t) = \sum_{n=1}^{\infty} b_n(t)\psi_n(x), \tag{51}$$

$$w(x, t) = \sum_{n=1}^{\infty} a_n(t)\psi_n(x),$$

where $\psi_n(x)$ are the even eigenfunctions of the free–free beam, λ_n being the associated eigenvalues, $n \geq 1$ [see Newman (1994)]. The eigenfunctions are assumed to be orthogonal. It is convenient to take the principal coordinates $a_n(t)$, $n = 1, 2, \ldots$ as the new unknown functions and to express with their help the coefficients $b_n(t)$ in equation (51).

The boundary condition on the elastic plate and expansions (51) provide

$$\varphi(x, y, t) = \sum_{m=1}^{\infty} \dot{a}_m(t)\phi_m(x, y),$$

where the functions $\phi_m(x, y)$ are the solutions of the boundary-value problems:

$$\Delta\phi_m = 0 \quad (y < 0), \qquad \phi_m = 0 \quad (y = 0, |x| > 1),$$

$$\partial\phi_m/\partial y = \psi_m(x) \quad (y = 0, |x| < 1),$$

$$\phi_m \to 0 \quad (x^2 + y^2 \to \infty).$$

We obtain

$$b_n(t) = \sum_{m=1}^{\infty} S_{nm}\dot{a}_m, \tag{52}$$

where

$$S_{mn} = \int_{-1}^{1} \phi_m(x, 0)\psi_n(x)\,\mathrm{d}x,$$

$S_{nm} = S_{mn}$ and $S_{11} = \pi/4$ for the rigid body mode. The coefficients S_{nm} have been found in their analytical forms with the help of the Bessel functions of zeroth and first order, the polynomial approximations for which are well-known.

The substitution of equations (51) and (52) into the beam equation (1) and the initial conditions (3) leads to the following system of ordinary differential equations:

$$\frac{\mathrm{d}\mathbf{a}}{\mathrm{d}t} = (\alpha_E I + S)^{-1}\mathbf{v},$$

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -D\mathbf{a} - \mathbf{q}(t) \qquad (53)$$

and the initial conditions

$$\mathbf{a}(0) = 0, \quad \mathbf{v}(0) = 0.$$

Here $\mathbf{a} = (a_1, a_2, \dots)^{\mathrm{T}}$, $\mathbf{v} = (v_1, v_2, \dots)^{\mathrm{T}}$, $v_n = \alpha_E \dot{a}_n + b_n$, $S = (S_{nm})_{n,m=1}^{\infty}$, $D = \mathrm{diag}\{0, \lambda_2^4, \lambda_3^4, \dots\}$, $\mathbf{q} = (q_1, q_2, \dots)^{\mathrm{T}}$, $q_n(t) = \int_{-1}^{1} q(x/l, t)\psi_n(x)\,\mathrm{d}x$, $\alpha_E = d/L$, I is the unit matrix.

Numerical calculations were performed for the initial-value problem (53) under the following conditions: $L = 20$ m, $d = 0{\cdot}72$ m, $E = 661{\cdot}7$ GPa, $h = 0{\cdot}8$ m, $T = 0{\cdot}4$ s, $\rho = 1025$ kg/m^3, $Q = 49$ kN/m^2, $q(x/l, t) = q_1(x/l)q_2(\omega t)$, $q_2(\tau) = 1 - |1 - 2\tau|$, where $0 < \tau < 1$, and $q_2(\tau) = 0$, where $\tau > 1$, $q_1(x/l) = [1 + \cos(\pi x/l)]/2l$, where $|x| < l$ and $q_1(x) = 0$ where $l < |x| < 1$. The total external load per unit width of the plate width is equal to 981 kN/m and does not depend on the parameter l at any time instant. A plate with the above characteristics but more than 10 times longer was used by Watanabe & Utsunomiya (1996) to simulate a Boeing 747 landing on a mat-like floating runway.

Watanabe & Utsunomiya (1996) examined the transient response of a pontoon-type floating structure due to impulsive loading. The finite element method was utilized to study fluid–structure interaction in the time-domain using three-dimensional linear theory. The original mat-like floating runway with a length of 3000 m, a width of 500 m, and a draft of 0·72 m was replaced by a circular plate with a radius of 200 m. It is believed that the size of 200 m should be enough for evaluating the dynamic response near the centre point, where the external concentrated load is applied. The circular elastic plate floats on the surface of an ideal liquid, which is 20 m deep and is bounded by a circular wall at a distance of 400 m from the plate centre. The problem is axi-symmetric. The parameter β is equal approximately to 0·00175, which implies that hydroelastic behaviour of this circular plate can be analysed by the method described in Section 4, where the case of very flexible floating plates is considered. The plate under consideration in the present section is much shorter and less flexible than that studied by Watanabe & Utsunomiya (1996). The landing loads are modelled by Watanabe & Utsunomiya (1996) as time-dependent impulsive loads of duration 0·4 s, which is the same as in the present paper, and 1 s. The largest period of vibrations of the circular plate floating on the liquid free surface can be estimated as of the order $O(T_w)$, where $T_w = \sqrt{\rho L^5/EJ}$, L being the plate radius here. In the case considered by Watanabe & Utsunomiya (1996), we obtain $T_w \approx 100c$, which is much greater than the duration of the external loads. Elastic effects and gravity effects cannot be decomposed as has been done in the present paper. Rigid displacement of the plate examined by Watanabe & Utsunomiya (1996) is less significant than that of the short plate considered in this section. Numerical results by Watanabe & Utsunomiya (1996) show that the dynamic effects of fluid coupling are of great importance. The decoupled problem is not considered in the present paper. In spite of the elastic characteristics of the plates being the same in the present analysis and that by Watanabe & Utsunomiya (1996), their geometrical characteristics are very different, which implies that they will have different hydroelastic behaviour.

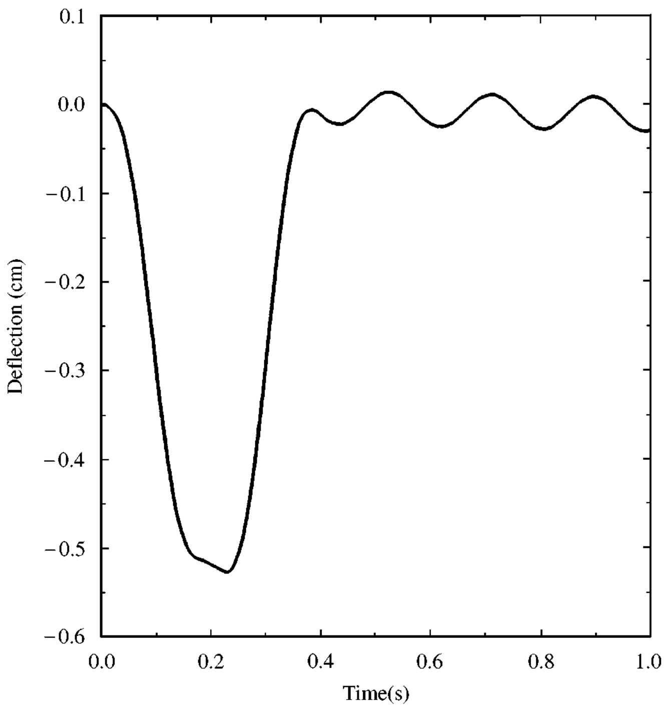

Figure 3. Elastic deflection at the plate centre.

In the present case $\beta = 17{\cdot}566$, $T_g = 1{\cdot}43$ s, $\omega = 0{\cdot}85$, $\alpha_E = 0{\cdot}036$ and $T_w = 0{\cdot}34$ s. It is important to notice that T_w is close to the load duration $T_q = 0{\cdot}4$ s and several times less than the time scale of gravity effects T_g. The scales of the beam deflection and strains are equal to 27·77 cm and 277·7 μs, respectively. The calculations were performed for the ratio of the external load area to the plate area l equal to 0·05, 0·1 and 0·5. The corresponding curves are very similar to each other. The results are shown for $l = 0{\cdot}1$ only.

Figures 3 and 4 show the elastic deflections of the plate at its mid-point and at the plate ends, respectively. It is seen that the deflections peak at the same time as the external load and are very small after the load is over. This behaviour of the elastic deflections is strongly connected with the fact that $T_w/T_q = O(1)$ and $T_w/T_g \ll 1$. For the same loads but for a longer plate, $T_w/T_q \gg 1$, we may expect that the elastic deflections peak after the cessation of the impact at an instant which is of the order of $O(T_w)$. It is of interest to note that elastic displacements of the beam end points are greater than the deflections at the beam mid-point. The rigid displacement of the plate is shown in Figure 5. Without account for gravity effects the rigid displacement of the plate continues to increase indefinitely with a velocity of about 0·27 m/s. This quantity corresponds well to the limiting value V_* defined in Section 5. We obtain $V_* \approx 0{\cdot}29$ m/s in the case under consideration. The plate continues to move down with approximately constant velocity after the cessation of the impact. This is a defect of the model used to describe the initial stage of the hydroelastic interaction between the short plate and the liquid. The solution of the model has to be considered as the initial asymptotics of the solution of the original problem as $t/T_g \ll 1$, and has to be matched to the approximate solution discussed in Section 5 for $t/T_g = O(1)$.

We may conclude that after a short initial stage, the duration of which is comparable with the duration of the external loads, a short plate can be treated as rigid (see Section 5). The

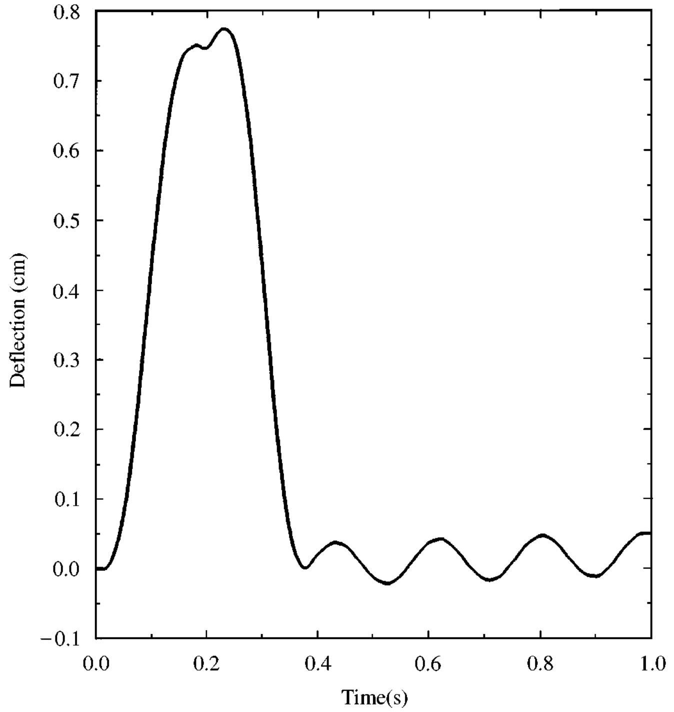

Figure 4. Elastic deflection at the plate edges.

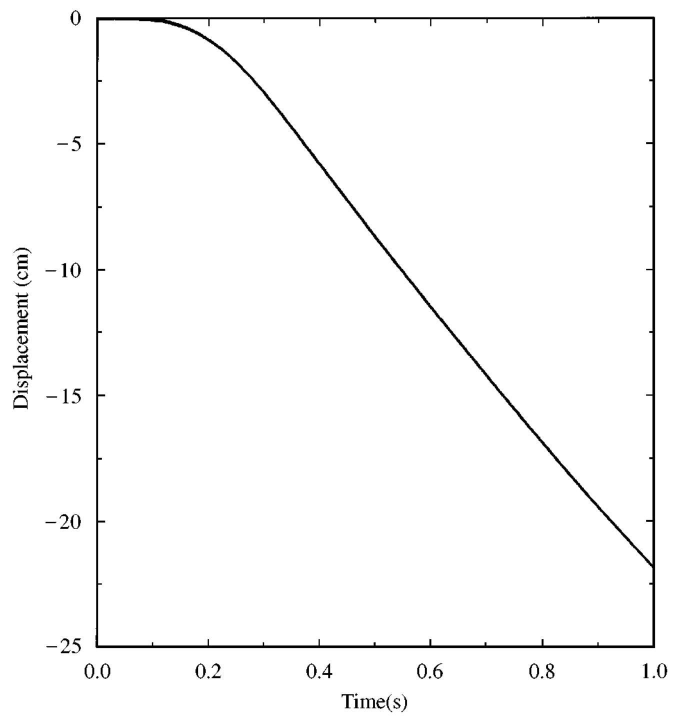

Figure 5. Rigid displacement of the plate due to impact.

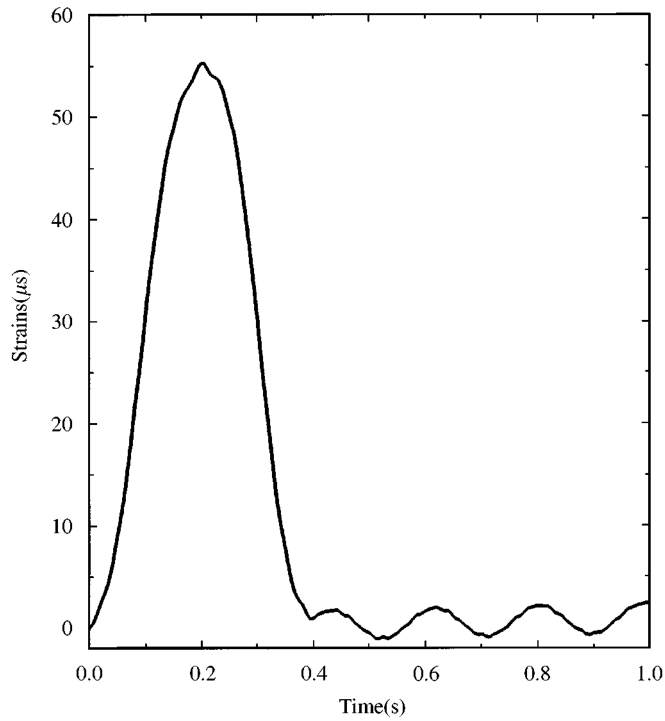

Figure 6. Strains at the plate centre.

evolution of the strains at the beam centre is depicted in Figure 6. Both the stresses and the deflections follow approximately the time history of the external loads. Strains at other points of the beam are smaller than at the centre point; for example, strains at distance 10 m from the centre are more than four times smaller (see Figure 7).

It is worth noticing that within the linear theory the deflections and strains are proportional to the magnitude of the external loads. Figures 4 and 5 indicate the possibility that the beam edges exit the water. Maximum absolute elevation of the beam edges occurs at the moment 0·1 s and is about 4·5 mm when the magnitude of the loads is 49 kN/m^2. The plate draft is 0·72 m, which means that loads having magnitudes of about 10 MN/m^2 may lead to lifting of the beam edges above the liquid surface.

It should be noted that, even during the impact stage, gravity effects are of importance close to the plate edges, where the present approach predicts infinitely large elevation of the free surface.

7. GENERAL THEORY

In the general case, the parameter β is of the order $O(1)$, which indicates that both the restoring and gravity forces balance the inertia force. We take T_g as the time scale and assume that elastic deflection of the plate and its rigid motion are of the same order of magnitude, $\kappa = 1$. In contrast to the approach outlined in Section 6, the parameter γ is unity now and the boundary condition (5) on the liquid free surface cannot be simplified.

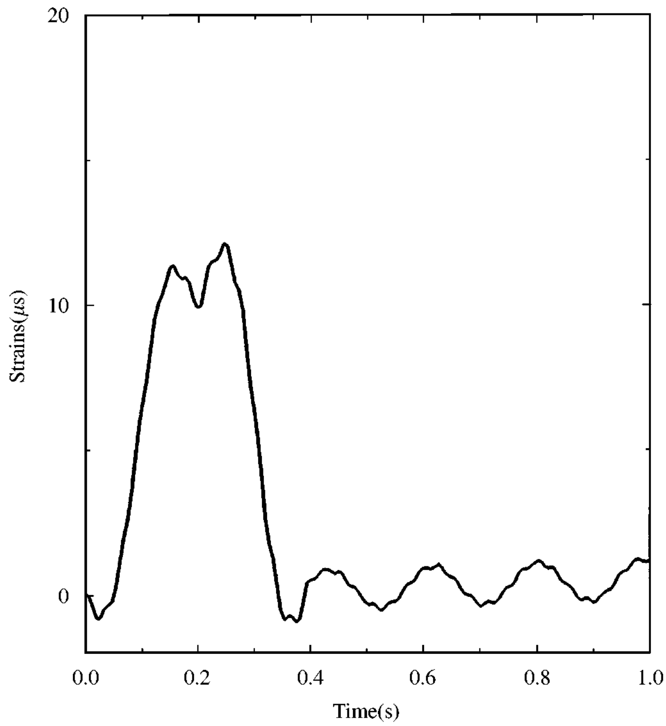

Figure 7. Strains at a distance of 10 m from the plate centre.

The velocity potential $\varphi(x, y, t)$ and the beam deflection $w(x, t)$ can be expressed as

$$\varphi(x, y, t) = \sum_{m=1}^{\infty} \left[\dot{a}_m(t)\phi_m(x, y) + \int_0^t \Phi_m(x, y, t-\tau)\dot{a}_m(\tau)\,\mathrm{d}\tau\right], \tag{54}$$

$$w(x, t) = \sum_{m=1}^{\infty} a_m(t)\psi_m(x),$$

where the functions $\phi_m(x, y)$ were defined in Section 6 and the response functions $\Phi_m(x, y, t)$ satisfy the following equations:

$$\Delta\Phi_m = 0 \quad (y < 0),$$

$$\frac{\partial^2 \Phi_m}{\partial t^2} + \frac{\partial \Phi_m}{\partial y} = 0 \quad (y = 0, |x| > 1), \tag{55}$$

$$\frac{\partial \Phi_m}{\partial y} = 0 \quad (y = 0, |x| < 1),$$

$$\Phi_m = 0, \quad \frac{\partial \Phi_m}{\partial t} = -\frac{\partial \phi_m}{\partial y} \quad (t = 0, y = 0, |x| > 1).$$

Substitution of equation (54) into the beam equation (1) and the initial conditions (3) gives the system of equations with respect to the principal coordinates $a_m(t)$, $m \geq 1$,

$$\frac{\mathrm{d}\mathbf{a}}{\mathrm{d}t} = (\alpha_E I + S)^{-1}\mathbf{v} + \int_0^t K(t-\tau)\mathbf{a}(\tau)\,\mathrm{d}\tau,$$

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -D\mathbf{a} - \mathbf{q}(t) \tag{56}$$

and the initial conditions

$$\mathbf{a}(0) = 0, \quad \mathbf{v}(0) = 0.$$

Here the vectors $\mathbf{a}$, $\mathbf{v}$ and $\mathbf{q}$ are the same as for system (53), where gravity effects were not taken into account. The diagonal matrix D is now given as $D = \text{diag}\{1, \beta\lambda_2^4 + 1, \beta\lambda_3^4 + 1, \ldots\}$ and the matrix $K(t)$ as $K(t) = (\alpha_E I + S)^{-1}\mu(t)$, where the elements $\mu_{mn}(t)$ of the matrix $\mu(t)$ are

$$\mu_{mn}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\int_{-1}^{1} \Phi_m(x, 0, t)\psi_n(x)\,\mathrm{d}x.$$

The initial-value problem for system (56) can readily be solved if the functions $\mu_{mn}(t)$, $m \geq 1$, $n \geq 1$, are known. These functions are independent of the parameters of the problem and can be considered as universal. Once the functions $\mu_{mn}(t)$ have been computed, they can be applied to any problem of floating plates.

8. CONCLUSION

The asymptotic and numerical analyses presented in this paper indicate that in some cases the description of hydroelastic behaviour of a floating plate can be essentially simplified. The approximate models derived can be used to estimate unknown values (deflections, strains, amplitude of heave motion, possibility of slamming effects and so on) and to clarify the mechanics of the phenomena. It is clear that after simple modifications the present analysis can be applied to three-dimensional problems with an arbitrary shape of the plate.

Concentrated loads were not considered here, but it is expected that asymptotic methods would be helpful to deal with this problem also. In the case of loads periodically distributed along a floating plate, where the wavelength of the loads is much less than the plate dimension, an averaging procedure can be used. The procedure shows that oscillations of external load inside the plate balance each other, and the main contribution to the plate motion comes from the periphery of the load area, where the hydrodynamic loads, deflections and stresses are concentrated. This conclusion is in line with the numerical results [see Kashiwagi (1998)] concerning the behaviour of floating plates in short waves.

ACKNOWLEDGEMENTS

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APPENDIX: NOMENCLATURE

$a(t)$	$= \dfrac{\partial^2 w^{(0)}}{\partial x^2}(1, t)$
$a_n(t)$	principal coordinates of the beam deflection
$\mathbf{a}$	$= (a_1, a_2, \ldots)^{\mathrm{T}}$
$b(t)$	$= \dfrac{\partial^3 w^{(0)}}{\partial x^3}(1, t)$

$b_n(t)$	principal coordinates of the velocity potential
d	plate draft
D	diagonal matrix in equation (53)
E	elasticity modulus
g	acceleration due to gravity
h	plate thickness
$\mathrm{H}(t)$	Heaviside function
I	unit matrix
J	inertia momentum of the beam cross-section
$K(t)$	$=(\alpha_E I + S)^{-1}\mu(t)$
L	a half of the plate length
l	ratio of external load area to the plate length
n, k, m, j	constants in equation (29)
P_{sc}	pressure scale
$p(x, y, t)$	hydrodynamic pressure
$p_i(\xi, \eta, t)$	hydrodynamic pressure in the "inner" variables
Q	magnitude of external loads
$q(x/l, \omega t)$	external load distribution
$q_n(t)$	coefficients in the expansion of external load distribution
$\mathbf{q}$	$=(q_1, q_2, \dots)^{\mathrm{T}}$
$s(t)$	rigid displacement
S_{sc}	scale of rigid displacement
S	matrix of hydrodynamic coefficients
S_{nm}	hydrodynamic coefficients
T	time scale
t	time
T_g	time scale for gravity effects
T_q	characteristic time of load duration
T_w	period of floating plate vibration
v_n	$=\alpha_E \dot{a}_n + b_n$
$\mathbf{v}$	$=(v_1, v_2, \dots)^{\mathrm{T}}$
W	scale of elastic deflections
$w(x, t)$	elastic deflection
$w_i(\xi, t)$	elastic deflection in the "inner" variables
$w^{(0)}, w^{(1)}$	coefficients in the deflection expansion (22)
$w_i^{(0)}, w_i^{(1)}$	coefficients in the deflection expansion (39)
x, y	Cartesian coordinates
α_E	$=\dfrac{\rho W d}{T^2 Q}$
α_R	$=\dfrac{\rho S_{sc} d}{T^2 Q}$
β	$=\dfrac{EJ}{\rho g L^4}$
β_0	$=\dfrac{EJW}{L^4 Q}$
γ	$=\dfrac{gT^2}{L}$
$\delta(t)$	Dirac delta-function
Δ	$=\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$
$\Delta w(t)$	elevation of the beam end points
κ	$=W/S_{sc}$
κ_E	$=\dfrac{LW}{T\Phi_{sc}}$
κ_R	$=\dfrac{LS_{sc}}{T\Phi_{sc}}$

λ	$=\dfrac{\rho\Phi_{sc}}{PT}$
λ_n	eigenvalues of free–free beam
$\mu(t)$	function defined in equation (47)
$\mu_{mn}(t)$	$=\dfrac{d}{dt}\displaystyle\int_{-1}^{1}\Phi_m(x,0,t)\psi_n(x)\,dx$
μ_E	$=\dfrac{\rho g W}{P_{sc}}$
μ_R	$=\dfrac{\rho g S_{sc}}{P_{sc}}$
v	$=\dfrac{P_{sc}}{Q}$
ξ, η	inner coordinates
ρ	liquid density
$\sigma(x,t)$	bending stresses
$\sigma_i(\xi,t)$	bending stresses in the "inner" variables
Φ_{sc}	scale of velocity potential
$\varphi(x,y,t)$	velocity potential
$\Phi(x,y,t)$	velocity potential in decomposition (45)
$\Phi_m(x,y,t)$	velocity potentials defined by equation (55)
$\varphi^{(0)}, \varphi^{(1)}$	coefficients in the potential expansion (21)
$\varphi_i(\xi,\eta,t)$	velocity potential in the "inner" variables
$\varphi_i^{(0)}, \varphi_i^{(1)}$	coefficients in the potential expansion (39)
$\phi_m(x,y)$	velocity potentials defined in section 6
$\psi(x,y)$	velocity potential in decomposition (45)
$\psi_n(x)$	even eigenfunctions of free–free beam
ω	$=\dfrac{T}{T_q}$

Special Notations

Dot stands for time derivative
Prime stands for dimensional variables

References

CUMMINS, W.E. 1962 The impulse response function and ship motions. *Schiffstechnik* **9,** 101–109.

KASHIWAGI, M. 1998 A B-spline Galerkin scheme for calculating the hydroelastic response of a very large floating structure in waves. *Journal of Marine Science and Technology* **3,** 37–49.

KOROBKIN, A.A. 1998 Numerical and asymptotic study of plane problem on hydroelastic behaviour of a floating plate in waves. *Journal of Applied Mechanics and Technical Physics* (submitted).

NEWMAN, J.N. 1994 Wave effects on deformable bodies. *Applied Ocean Research* **16,** 47–59.

STUROVA, I.V. 1998 The oblique incidence of surface waves onto the elastic band. In *Proceedings of the Second International Conference on Hydroelasticity in Marine Technology* (eds M. Kashiwagi, W. Koterayama & M. Ohkusu), pp. 239–245, Fukuoka, Japan, 1–3 December 1998.

WATANABE, E. & UTSUNOMIYA, T. 1996 Transient response analysis of a VLFS airplane landing. In *Proceedings of International Workshop on Very Large Floating Structures,* Ship Research Institute, pp. 243–247.

WAGNER, H. 1932 Über Stoss- und Gleitvorgänge an der Oberfläche von Flüssigkeiten. *Zeitschrift für Angewendte Mathematik und Mechanik* **12,** 192–235.

WU, C., WATANABE, E. & UTSUNOMIYA, T. 1995 An eigenfunction expansion-matching method for analyzing the wave-induced responses of an elastic floating plate. *Applied Ocean Research* **17,** 301–310.

YAGO, K. & ENDO, H. 1996 Model experiment and numerical calculation of hydroelastic behaviour of matlike VLFS. In *Proceedings of International Workshop on Very Large Floating Structures,* Ship Research Institute, pp. 209–216.